

# Suppression of superconductivity due to non-perturbative saddle points in the nonlinear $\sigma$ -model

D. A. Pesin and A. V. Andreev

Department of Physics, University of Washington, Seattle, WA 98195, USA

We study superconductivity suppression due to thermal fluctuations in disordered wires using the replica nonlinear  $\sigma$ -model ( $NL\sigma M$ ). We show that in addition to the thermal phase slips there is another type of fluctuations that result in a finite resistivity. These fluctuations are described by saddle points in  $NL\sigma M$  and cannot be treated within the Ginzburg-Landau approach. The contribution of such fluctuations to the wire resistivity is evaluated with exponential accuracy. The magnetoresistance associated with this contribution is negative.

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Suppression of superconductivity in low dimensional samples has been studied for a long time. In particular, in quasi-one-dimensional samples supercurrent decay at temperature,  $T$ , below and near the critical temperature,  $T_c$ , is believed to occur via thermally activated phase slips (TAPS). This idea was put forward by Little [1], and the quantitative theory was developed by Langer, Ambegaokar, McCumber and Halperin (LAMH) [2, 3]. Within the LAMH theory the wire resistance below  $T_c$ ,  $R$ , has activated behavior,  $R \propto \exp(-\Delta F/T)$ , where  $\Delta F$  is the activation energy for a TAPS.

Most experimental results appear to be in good agreement with the LAMH theory [4, 5]. However some discrepancies have also been reported [6, 7]. In particular, the *negative* magnetoresistance observed in thin Pb wires in Ref. [6] has no explanation within the LAMH theory.

From a theoretical viewpoint, the LAMH theory relies on the Ginzburg-Landau description of thermal fluctuations resulting in finite resistivity. In this paper we study superconductivity suppression in disordered wires using a more general description of a superconductor in terms of the replica nonlinear  $\sigma$ -model ( $NL\sigma M$ ) [8] that includes both the pairing and the Coulomb interaction. This formalism describes the low energy physics in terms of the order parameter,  $\Delta$ , the fluctuating electric potential,  $V$ , and the  $Q$ -matrix that describes the fluctuating elec-

tron Green function. Since the Ginzburg-Landau functional can be obtained [9] from the  $NL\sigma M$  the LAMH optimal fluctuation is automatically taken into account within this formalism and appears as a saddle point of the  $NL\sigma M$  action.

We show that due to the interplay of Coulomb interaction and disorder there exist different saddle points of the  $NL\sigma M$  action that provide alternative channels for supercurrent decay. These fluctuations cannot be described within the Ginzburg-Landau approach. They can be understood as follows. The interplay of disorder and Coulomb interaction leads to soliton-like fluctuations of the  $Q$ -matrix analogous to those first studied in Ref. [10] in the context of charge quantization in a single electron box with diffusive contacts. In a disordered wire these solitons are accompanied [11] by a change of the *imaginary* part of the electric potential by  $2\pi T$  on the scale of thermal diffusion length,  $L_T = \sqrt{D/2\pi T}$ , where  $D$  is the diffusion constant. Below  $T_c$  the Coulomb and pairing interactions result in bound soliton-antisoliton pairs with a typical spatial separation,  $L_c \sim \min\{L_N, L_S\}$ , with  $L_N \sim \frac{e^2}{2\pi^2 T} \ln \frac{e^2}{2\pi^2 T d}$  and  $L_S \sim [\nu(T_c - T)\Delta_0^2 A/T_c^2]^{-1}$ , where  $\nu$  is the density of states at the Fermi level,  $A$  is the wire cross section area,  $\Delta_0$  is the equilibrium value of the superconducting gap,  $e$  is the electron charge, and  $d$  is the transverse dimension of the wire. In the vicinity of the critical temperature  $L_c \gg L_T$ , and the soliton and antisoliton within each pair are well separated. We show below that superconductivity is suppressed in the middle of the soliton-antisoliton pair, see Fig. 1. This results in an additional wire resistance

$$\delta R(T) = R_0 \exp[-2G(L_T) - 2\Delta F/T]. \quad (1)$$

Here  $\Delta F/T \approx 0.35 G(L_T)[(T_c - T)/T_c]^{3/2}$  is the activation action of the LAMH phase slip, with  $G(L_T) \equiv 4\pi\nu DA/L_T \gg 1$  being the dimensionless conductance of the wire segment of length  $L_T$ . Equation (1) is valid for  $G(L_T)^{-2/3} \ll (T_c - T)/T_c \ll G(L_T)^{-1/2}$ , where the first inequality corresponds to the Ginzburg criterion and the second ensures the condition  $L_c \gg L_T$ .

Although rigorous evaluation of the pre-exponential

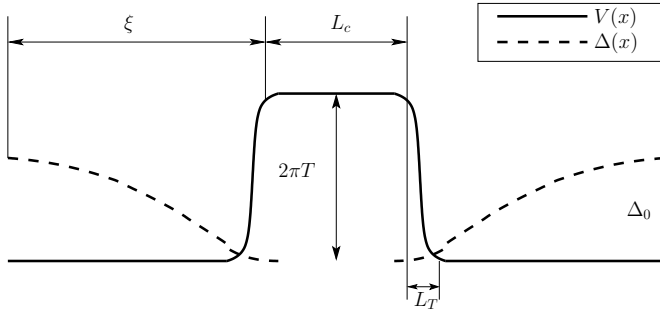


FIG. 1: Sketch of the electric potential,  $V(x)$ , (solid line) and superconducting gap,  $\Delta(x)$ , (dashed line) dependence on the coordinate  $x$  along the wire at the saddle point configuration.

factor,  $R_0$ , is rather difficult and left for future work, we speculate that the interior of each soliton-antisoliton pair acts as a segment of a normal metal, see discussion below Eq. (10). This reasoning leads to the estimate  $R_0 \simeq R_N \frac{L_T^2}{L^2}$ , where  $R_N$  is the wire resistance in the normal state.

The magnetoresistance associated with the contribution of these saddle points is negative at low fields and is given by Eq. (12). This may be relevant to the negative magnetoresistance observed in Ref. [6].

We consider a long disordered wire with many transverse channels. The temperature dependent coherence length  $\xi(T)$  is assumed to exceed the transverse wire dimension,  $d$ . The Thouless energy for the transverse motion,  $E_T \equiv D/d^2$ , satisfies the condition  $E_T \gg T$ . In this regime the wire is described by a one-dimensional  $NLM$ .

We write the replicated partition function for the system,  $\langle Z^n \rangle = \langle e^{-n\frac{\hat{H}}{T}} \rangle$ , with  $n$  being the number of replicas, as an imaginary time functional integral over the fermions. Upon disorder averaging, the  $Q$ -matrix (in the space of replicas and Matsubara frequencies) is introduced to decouple the disorder-induced quartic interaction. Its matrix elements are  $4 \times 4$  matrices in the space  $S \otimes T$ , given by the product of spin,  $S$ , and time-reversal,  $T$ , spaces [12, 13]. The auxiliary fields  $V_a(x, \tau)$  and  $\Delta_a(x, \tau)$  decouple the Coulomb and pairing interactions in the replica  $a$  via Hubbard-Stratonovich transformation. Proceeding in the usual way [8], one arrives at the following form of the action for  $Q$ ,  $V$  and  $\Delta$ :

$$\langle Z^n \rangle = \int \mathcal{D}[Q, V, \Delta] e^{-S_Q - S_\Delta - S_C}, \quad (2a)$$

$$S_Q = A \frac{\pi\nu}{2} \int dx \text{Tr} \left[ \frac{D}{4} (\nabla Q)^2 - (\hat{\varepsilon} + \hat{V} + i\hat{\Delta})Q \right] + A\nu \int d\tau dx \sum_a V_a^2(x, \tau), \quad (2b)$$

$$S_\Delta = A \int dx d\tau \sum_a \frac{|\Delta_a(x, \tau)|^2}{\lambda}, \quad (2c)$$

$$S_C = \frac{1}{2} \int d\tau dx dx' \sum_a V_a(x, \tau) C(x, x') V_a(x', \tau). \quad (2d)$$

Here  $\text{Tr}$  denotes the trace over the replica, Matsubara and  $S \otimes T$  spaces. The matrices  $\hat{\varepsilon}$ ,  $\hat{V}$  and  $\hat{\Delta}$  have the following structure in the replica and  $S \otimes T$  spaces:  $\hat{\varepsilon} = i\delta^{ab}\tau_3\partial_\tau$ ,  $\hat{V} = \delta^{ab}\tau_0 V_a$ ,  $\hat{\Delta} = \delta^{ab}(\Delta_a\tau_+ - \Delta_a^*\tau_-)$ , where  $\tau_\pm = \frac{\tau_1 \pm i\tau_2}{2}$  and  $\tau_i$ 's are defined as  $\tau_i = t_i \otimes \sigma_0$ , with  $\sigma_i$  and  $t_i$  being the Pauli matrices in the  $S$  and  $T$  spaces respectively. The positive constant  $\lambda$  denotes the attraction strength in the Cooper channel, and  $C(x, x')$  describes the inverse effective Coulomb interaction in the wire. In particular, for a homogeneous wire in the absence of a nearby gate its Fourier component is  $C(q) = 1/e^2 \ln \frac{1}{q^2 d^2}$ .

The saddle point equations are obtained by minimizing the action in Eq. (2a) with respect to  $\Delta$ ,  $V$  and  $Q$ . For

this purpose one may neglect  $S_C$  in (2) if the number of channels in the wire is sufficiently large,  $e^2\nu A \gg 1$ . This corresponds to the charge neutrality limit [11] and gives:

$$D\nabla(Q\nabla Q) - [\hat{\varepsilon} + \hat{V} + i\hat{\Delta}, Q] = 0, \quad (3a)$$

$$V_a - \frac{\pi}{4} \text{tr} Q_{\tau\tau}^{aa}(x) = 0, \quad (3b)$$

$$\Delta_a + i\frac{\pi\nu\lambda}{4} \text{tr}[\tau_- Q_{\tau\tau}^{aa}(x)] = 0, \quad (3c)$$

where  $\text{tr}$  is the trace in the  $S \otimes T$  space. Equation (3a) is the Usadel equation, Eq. (3b) represents the charge neutrality condition, and Eq. (3c) plays the role of the BCS self-consistency equation.

Before discussing the superconducting case, let us briefly describe solutions of the saddle point equations (3) in a normal metal,  $\Delta_a = 0$  [11]. Equations (3a) and (3b) possess a set of degenerate stationary spatially uniform solutions, where  $Q_{nm}^{ab} = \delta^{ab}\delta_{nm}\tau_0 \text{sgn}(\varepsilon_n + 2\pi T w_a)$ ,  $V_a = 2\pi T w_a$  with  $w_a$  being an integer,  $n, m$  being the Matsubara indices, and  $\varepsilon_n = \pi T(2n+1)$ . The sum  $4 \sum_a w_a \equiv W$  (the factor 4 here arises from the  $4 \times 4$  structure in the  $S \otimes T$  space) defines the trace of the  $Q$ -matrix,  $\text{Tr} Q = 2W$ . Different  $W$ 's define topological classes of  $Q$ , i.e.  $Q$  matrices corresponding to the same  $W$  can be continuously rotated one into another in the replica and Matsubara spaces. Thus in addition to the above spatially uniform saddle point configurations, the action (2) has spatially inhomogeneous finite-action solitons as saddle configurations. These solitons connect different  $Q$ 's of the same topological class, and correspond to both  $Q$  and  $V_a$  changing in space. These nonuniform saddle points are crucial for the subsequent considerations.

Let us construct the soliton that connects  $Q_{nm}^{ab} = \Lambda_{nm}^{ab} \equiv \delta^{ab}\delta_{nm}\tau_0 \text{sgn}\varepsilon_n$  at  $x = -\infty$  and  $Q_{nm}^{ab} = \delta^{ab}\delta_{nm}\tau_0 \text{sgn}(\varepsilon_n - 2\pi T w_a)$  at  $x = \infty$ , with  $w^{1,2} = \mp 1$ , and all the other  $w$ 's are zero. This corresponds to gradual change in the electric potential in replicas 1 and 2 from zero at negative infinity to  $\mp 2\pi iT$  at positive infinity. For simplicity, we assume strong spin-orbit scattering and consider the  $Q$ -matrix belonging to the symplectic ensemble [12]. In this case, the soliton  $Q$ -matrix can be parameterized by a single rotation angle  $\theta(x)$  [10]:

$$Q_N(x) = e^{-i\frac{\theta_i(x)}{2}\hat{u}\otimes\tau_i}\Lambda e^{i\frac{\theta_i(x)}{2}\hat{u}\otimes\tau_i}, \quad (4)$$

where  $\hat{u}$  is the generator of rotation between the Matsubara frequencies  $\pi T$  in replica 1 and  $-\pi T$  in replica 2,  $u_{nm}^{ab} = \delta^{a1}\delta^{b2}\delta_{n,0}\delta_{m,-1} + \delta^{a2}\delta^{b1}\delta_{n,-1}\delta_{m,0}$ . The index  $i = 0, 1$  labels diffuson-like,  $\tau_0$ , and cooperon-like,  $\tau_1$ , rotations. Substitution of Eq. (4) into (3b) gives  $V_{1,2}(x) = \mp \pi T[1 - \cos\theta(x)]$ . Then Eq. (3a) gives

$$\nabla^2\theta_i - \frac{1}{2L_T^2} \sin 2\theta_i = 0. \quad (5)$$

This equation has a solution  $\theta_i(x) = 2 \arctan(e^{x/L_T})$ , which gives for the electric potentials  $V_{1,2}(x) = \mp \pi T[1 +$

$\tanh(x/L_T)$ ]. The action (2b) of this soliton is  $G(L_T)$ . It can be shown that this saddle point is stable [14].

In a macroscopic system, solutions with nonzero  $w_a$ 's at  $x \rightarrow \pm\infty$  have infinite Coulomb action (2d) and thus are forbidden. Inhomogeneous saddle points appear as bound soliton-antisoliton pairs with  $V_a(x)$  appreciably different from zero only in the interior of the pairs, see Fig. 1. In the regime of interest,  $G(L_T) \gg 1$ , the Coulomb action (2d) does not affect the shape of the solitons, but merely determines the typical soliton-antisoliton separation,  $L_N$ . The latter can be estimated by equating the Coulomb action of a pair,  $S_C \sim \frac{2\pi^2 T L_N}{e^2 \ln(L_N/d)}$ , to unity, resulting in  $L_N \sim \frac{e^2}{2\pi^2 T} \ln \frac{e^2}{2\pi^2 T d}$ . We assume that  $L_N \gg L_T$ . This assumption corresponds to  $T\tau_{el} \ll \frac{3}{2\pi} (\frac{e^2}{v_F})^2 \ln^2 \frac{e^2}{\pi^2 T d}$  ( $\tau_{el}$  is the mean free time) and is always satisfied in the  $NL\sigma M$  applicability region. Since the soliton and antisoliton in a typical pair are well separated, the pair action cost is  $2G(L_T)$ .

We now turn to the superconducting case. At  $T > T_c$  Eq. (3c) implies  $\Delta_a = 0$ , and the solutions of Eqs. (3a), (3b) remain the same as in the normal case. Below  $T_c$ , in the region of interest,  $T_c - T \ll T_c$ , a small static superconducting order parameter  $\Delta_a$  appears. The saddle point solution,  $Q_S$ , can be expressed in terms of the normal state solution (4) as  $Q_S = U^\dagger Q_N U$ , where  $U$  is a replica-diagonal rotation matrix in the Gorkov-Nambu space,  $U = \delta^{ab} U^a$ , with

$$U_{nm}^a = \delta_{n,m} \cos \frac{\phi_n^a}{2} - \tau_2 e^{-i\tau_3 \chi_a} \delta_{n, -(m+1)} \text{sgn}(\varepsilon_n) \sin \frac{\phi_n^a}{2}. \quad (6)$$

Here  $\chi_a$  are the order parameter phases.

Solving Eq. (3a) using the ansatz (6) with the spatially uniform  $Q_N = \Lambda$ , and substituting the result into (3c) one recovers the Ginzburg-Landau equation for the order parameter, which contains both the homogeneous BCS solution and the LAMH saddle point.

Let us turn to the inhomogeneous  $Q_N$ , Eq. (4). In the replicas  $a \neq 1, 2$  the situation is the same as for  $Q_N = \Lambda$ . For  $a = 1, 2$  substitution of the ansatz (6) into Eqs. (3) leads to the following equations for  $\phi_n^a$ , and  $\Delta_a$ ,

$$\frac{1 + \cos \theta}{2} \nabla^2 \phi_0^a + \nabla \cos \theta \nabla \phi_0^a = \frac{\sin \phi_0}{L_T^2} - \frac{2\Delta_a}{D} \cos \phi_0^a, \quad (7a)$$

$$\nabla^2 \phi_n^a = \frac{2\varepsilon_n}{D} \sin \phi_n^a - \frac{2\Delta_a}{D} \cos \phi_n^a, \quad (7b)$$

$$\frac{\Delta_a}{2\pi\nu\lambda T} = \sum_{n>0} \sin \phi_n^a + \frac{1 + \cos \theta}{2} \sin \phi_0^a. \quad (7c)$$

Here we assumed that there is no current in the wire and set the order parameter phase,  $\chi$ , to zero. The additional equation for  $\theta$  at  $T_c - T \ll T_c$  may be approximated by Eq. (5). Therefore below we consider  $\theta$  in Eqs. (7) to be the (approximate) solution of Eq. (5) corresponding to the soliton-antisoliton pair in the normal metal. Without loss of generality we consider the soliton and antisoliton to be positioned at  $x = \pm\zeta$  with  $\zeta \sim L_c \gg L_T$ ,

where  $L_c$  is the typical soliton-antisoliton separation in the presence of superconductivity, estimated below.

The exact solution of the system of equations (7) is prohibitively difficult. However, near the critical temperature the situation simplifies. Outside the soliton-antisoliton pair,  $|x| > \zeta$  the GL approach may be used. From the viewpoint of the GL description the presence of the soliton-antisoliton pair imposes a boundary condition on the order parameter  $\Delta_a$  at  $x = \pm\zeta$ ,

$$\nabla \ln \Delta_a(x)|_{x=\pm\zeta} = \pm\kappa, \quad (8)$$

where  $\kappa \sim L_T^{-1} \ll \xi(T)^{-1}$ . The form of this boundary condition can be obtained from the following considerations. The effective ‘‘critical temperature’’,  $T_c^*$ , for the appearance of a static order parameter,  $\Delta_a$ , is lowered inside the soliton-antisoliton pair. Indeed, for  $\cos \theta = -1$  linearizing Eqs. (7b) and (7c) for a uniform static  $\Delta_a$  we obtain

$$\frac{1}{2\pi\nu\lambda} = T_c^* \sum_{n>0} \frac{1}{\varepsilon_n}, \quad (9)$$

where as usual the sum over Matsubara frequencies in the r.h.s. must be cut off at the Debye frequency. The restriction  $n > 0$  in the summation arises because the last term in Eq. (7c) vanishes for  $\cos \theta = -1$ . Expressing the l.h.s. of Eq. (9) in terms of  $T_c$  we find  $T_c^* = T_c/e^2$ , where  $e \approx 2.71$  is the base of the natural logarithm. Thus the regime of interest,  $T_c - T \ll T_c$ , corresponds to the ‘‘normal’’ phase inside the soliton-antisoliton pair [15]. Since  $(T - T_c^*)/T_c^* \sim 1$  the effective correlation length inside the pair is of order  $L_T$ , and the order parameter present outside the pair penetrates into its interior to distances of order  $L_T \ll \xi(T)$ . This leads to the boundary condition (8) similar to the proximity effect for a normal metal in contact with a superconductor [16]. Then in the limit  $L_T \ll \xi(T)$ ,  $h$  the boundary condition (8) may be replaced by  $\Delta_a(\pm\zeta) = 0$ , and the order parameter  $\Delta_a(x)$  acquires the form,

$$|\Delta_a(x)| = \Delta_0 \Theta(|x| - \zeta) \tanh \frac{|x| - \zeta}{\sqrt{2}\xi(T)}, \quad (10)$$

where  $\Theta(x)$  is the step function.

Since the order parameter profile outside the soliton-antisoliton pair coincides with that in the LAMH fluctuation in the zero current limit the action cost of such a configuration is  $S_0 = 2G(L_T) + 2\Delta F/T$ , where  $\Delta F$  is the LAMH free energy barrier in the limit of zero bias current. The factor 2 in front of  $\Delta F$  arises because the condensate suppression occurs in two replicas. In this estimate we neglected the Coulomb action and the condensation free energy in the interior of the pair,  $S_\zeta \sim \frac{2\pi^2 T \zeta}{e^2 \ln(\zeta/d)} + \zeta A \nu (T_c - T) \Delta_0^2 / T_c$ . For a typical fluctuation  $S_\zeta \sim 1 \ll S_0$ . This gives for a typical fluctuation size,  $L_c \sim \min\{L_N, L_S\}$ , where  $(L_S)^{-1} = \nu(T_c - T) \Delta_0^2 A / T_c^2$ .

With exponential accuracy the contribution of such fluctuations to the wire resistance is determined by their action, which results in Eq. (1). Evaluation of the pre-exponential factor  $R_0$  is a difficult task. It may in principle be accomplished using linear response theory. This involves a cumbersome procedure of analytic continuation of the Matsubara susceptibility to real frequencies and is outside the scope of the present work. It is nevertheless plausible that the interior of the soliton-antisoliton pair acts as a segment of a normal metal [17], giving a contribution  $\frac{h}{e^2 G(L_T)} \frac{L_c}{L_T}$  to the wire resistance. The number of soliton-antisoliton pairs in the wire can be estimated as  $\frac{L L_c}{L_T^2} e^{-S_0}$ , where the prefactor corresponds to  $L/L_T$  ways to accommodate a soliton, and  $L_c/L_T$  possibilities to put an antisoliton next to it. Following this argument one arrives at the estimate  $R_0 \simeq R_N \frac{L_c^2}{L_T^2}$ .

We now discuss the magnetic field dependence of resistance (1). It arises from the decrease of condensation energy  $\Delta F$  due to magnetic field, and an increase of the action of cooperon-like solitons. In the presence of a magnetic field,  $H$ , the  $NL\sigma M$  action for a wire can be obtained [13] by changing  $(\nabla Q)^2 \rightarrow (\nabla Q)^2 - e^2 \langle \mathbf{A}^2 \rangle [\tau_3, Q]^2$  in Eq. (2b), where  $\langle \mathbf{A}^2 \rangle$  is the square of the vector potential,  $\mathbf{A}$ , averaged over the wire cross-section (we assume that the magnetic length,  $l_H = \sqrt{\frac{1}{eH}}$ , is much larger than the wire width,  $d$ ). Each soliton or antisoliton constituting a pair can be generated either by a diffuson-like or a cooperon-like rotation,  $i = 0, 1$  in Eq. (4). The diffuson is insensitive to the magnetic field whereas the cooperon acquires a mass. As a result, Eq. (5) for the cooperon rotation angle on the soliton configuration becomes

$$\nabla^2 \theta_c - \frac{1}{2} \left( \frac{1}{L_T^2} + \frac{1}{L_H^2} \right) \sin 2\theta_c = 0, \quad (11)$$

where  $L_H^{-2} = e^2 \langle \mathbf{A}^2 \rangle \propto H^2$  depends on the specific geometry, in particular for a rectangular wire of width  $d$ , and a magnetic field perpendicular to it,  $L_H^2 = 3l_H^4/d^2$ . The corresponding action change for a single cooperon-like soliton is  $G(L_T) \left( \sqrt{1 + L_T^2/L_H^2} - 1 \right)$ . The change in the condensation action,  $\Delta F/T$ , may be evaluated using the Ginzburg-Landau theory and is much smaller, of the order of  $G(L_T) \frac{L_T^2}{L_H^2} \sqrt{(T_c - T)/T_c}$ . At relatively weak fields,  $L_H^2 \gtrsim G(L_T) L_T^2 \gg L_T^2$ , it may be neglected while the action change for the cooperon-like soliton can be significant. In this regime we obtain [18]

$$\delta R(H) \approx \frac{\delta R(T)}{4} \left[ 1 + e^{-G(L_T) L_T^2 / 2 L_H^2} \right]^2. \quad (12)$$

Here  $\delta R(T)$  is given by Eq. (1) and we took into account that there are four ways to assemble a soliton-antisoliton pair out of diffuson-like and cooperon-like solitons. From

Eq. (12) we see that the resistance (1) decreases with magnetic field.

In summary, we have considered superconductivity suppression in thin wires due to non-perturbative saddle points in the  $NL\sigma M$ . We showed that their presence leads to an additional (to the LAMH) contribution to the wire resistance below  $T_c$  and evaluated it with exponential accuracy. This contribution is suppressed by a magnetic field, which may correspond to the negative magnetoresistance observed in Ref. [6].

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  - [15] Condensation at nonzero Matsubara frequencies,  $\Delta(\tau) \propto \exp(i4\pi w_a \pi T \tau)$ , would occur at  $T = T_c$  in the homogeneous case. Since the typical soliton-antisoliton separation  $L_c$  is much smaller than  $\xi(T)$ , see discussion below Eq. (10), the non-stationary condensates do not appear.
  - [16] A. A. Abrikosov, Fundamentals of the Theory of Metals (Elsevier, Amsterdam, 1988)
  - [17] In principle, the soliton-antisoliton pair may be viewed as an SNS junction, but because its size,  $L_c$  exceeds the coherence length in the interior, which is of the order of  $L_T$ , the expected critical current is exponentially small.
  - [18] We neglect the magnetic field dependence of the pre-exponential factor. We believe this is justified for  $L_H \gg L_T, \xi(T)$ .